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TECHNICAL NOTE

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AN EMERGENCY MIDCOURSE NAVIGATION PROCEDURE FOR

A SPACE VEHICLE RETURNING FROM THE MOON

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SUMMARY

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A manual emergency navigation procedure for a vehicle returning from the moon is presented. The procedure involves photographing the earth from various positions along the trajectory returning from the moon and deducing from image measurements the required midcourse corrections. Preliminary tests of the accuracy with which required image measurements can be obtained together with machine computations of the over-all accuracy of the guidance procedure indicate the accuracy could be within the corridor depth of a lifting entry vehicle.

INTRODUCTION

The purpose of this paper is to present some results of a manual navigation procedure which could be used as an emergency guidance system. The procedure is an example of designing a system for simplicity by giving a pilot the minimum instrumentation necessary to accomplish the mission. A back-up system must be designed to function under a wide range of emergency conditions. To cover as wide a range of emergency conditions as possible, it was assumed in this investigation that no electronic computing or sensing equipment was available. With such equipment eliminated, the only apparent navigational instruments are optical devices. This investigation considers the use of photographic devices in preference to other optical devices, because the more deliberate manner in which a photograph can be examined should result in greater operational reliability.

A specific mission has been assumed to illustrate the application of this manual guidance procedure. Of primary concern is the safe return from the vicinity of the moon into the earth's atmosphere, and of secondary interest is reentry at some specified geographic longitude. Limiting the present investigation to this mission should not eliminate the procedure from consideration for different missions, but a feasibility study would be necessary for other specific missions.

The navigation procedure considered is based on primary information received only from photographs and a clock. The procedure involves the computation of two-body orbital parameters and required velocity corrections from measurements of photographic images of the earth against a star background. Details concerning these photographs, and the associated calculations, are discussed in appendices to the report. The investigation includes tests to determine the accuracy to which typical images of the earth might be made, and machine computations to show the over-all accuracy of the navigational procedure.

NOTATION

A	orbital semimajor axis, ft
d, D	film measurements defined in appendix A, in.
e	orbital eccentricity
f	fraction of computed velocity correction actually applied
H	orbital angular momentum per unit mass, ft^2/sec
l	film measurement defined in appendix A, in.
R	distance of vehicle from mass center of earth, ft
$\dot{R}$	$\left(\frac{dR}{dt}\right)$, ft/sec
R_e	maximum optical radius of earth, ft
R_{per}	orbital perigee radius, ft
Δt	time interval between points 1 and 3 (fig. 1), sec
ΔT	time interval between points 2 and 4 (fig. 1), sec
V	orbital velocity, ft/sec
α	angles defined in appendix A, radians
$\epsilon_{R_{max}}$, $\epsilon_{C_{max}}$	error quantities defined in figure 4, in.
θ	orbital angular position, radians
$\dot{\theta}$	$\left(\frac{d\theta}{dt}\right)$, radians/sec
τ	time for vehicle to reach perigee, sec
μ	gravitational constant for earth, 1.40773×10^{16} ft^3/sec^2

Subscripts

1, 2, 3, 4 designates points in figure 1

PROPOSED METHOD OF NAVIGATION

A pilot can place his vehicle in a safe braking orbit if he can determine the vacuum perigee radius of the actual orbit and apply the velocity corrections necessary to achieve an orbit with a safe perigee radius. In the proposed method considered in this report, the pilot obtains the primary information required by photographing the earth against the star background at two points along the trajectory and then measuring the time increment between taking the two photographs.

The typical lunar return trajectory shown in figure 1 is, outside the relatively short range of influence of the moon, essentially a Keplerian ellipse with an eccentricity of about 0.95. Points 1 and 3 represent locations where photographs are taken in the navigation procedure. The two photographs would have the same optical magnification and contain the earth's image with at least two of the same stars appearing in the background of both photographs. The diameter of the earth images on these photographs is measured with the simple optical devices shown in figure 2. A transparent overlay containing concentrically inscribed circles with accurately known diameters is centered over the earth image, and a small inscribing tool is used to mark the image center through a tightly fitting sleeve in the center of the transparent overlay. The image diameter is measured with a simple shop microscope, measuring the distance of the image edge from the nearest inscribed circle. (The shop microscope is also used to center the image by making this distance equal on opposite edges of the image.) After the diameters are measured, one photograph is placed on top of the other with the two common background stars superimposed and the distance between image centers is measured using the transparent overlay and shop microscope as shown in figure 3. This distance is related directly, through the optical magnification, to the angle $\Delta\theta$ shown in figure 1. Given the range at points 1 and 3, the value of $\Delta\theta$, and the time increment between the two points, one can compute approximate values for range, range rate, and angular rate for point 2. The two-body equations are then used to compute the orbital perigee radius and to obtain the necessary correction velocity. These computations are discussed in more detail in appendix A.

Provision must be made to orient the vehicle manually both for taking photographs and for firing the course correction rockets; low thrust jets could be used to cancel angular velocities, and, if an inertial wheel is not available, a crew member circling the cabin walls could supply the desired angular positioning. The thrust of the course correction rockets would be directed in the orbital plane perpendicular to the radius vector to earth. Such alinement should be fairly simple since observations of the earth's motion against the star field will define the orbital plane relative to the celestial sphere.

To increase the accuracy of the first velocity correction, the average value of measurements and calculations from about 10 photographs should be determined. Also, an additional set of calculations may be performed to permit a coarse adjustment of the time at which the vehicle will arrive at perigee radius. This time can be adjusted so that the vehicle will enter the atmosphere at a desired geographic longitude. For purposes of discussion, this procedure will be referred to as the "initial correction" procedure as opposed to the "general correction" procedure which uses only two photographs. The initial correction procedure is discussed in detail in appendix B.

TEST PROCEDURES

Optical Measurements

In order to estimate the accuracy with which photographic measurements can be performed manually, a series of photographs were taken, and measurements were made in a manner similar to that postulated for the emergency navigation scheme proposed. A white disk was mounted on a black background and photographed from various distances to give photographic images ranging from 3 to 4 inches in diameter. The disk was inclined slightly to give varying amounts of ellipticity corresponding to the earth's ellipticity as seen from different positions in space.

Two devices provided accurate measurement of the photographic images: a transparent overlay with accurately inscribed circles, and a small 30-power shop microscope. The inscribed circles on the transparent overlay were drawn with a radial separation of 0.1 inch and a line weight of about 0.002 inch. The smallest division of the shop microscope was 0.001 inch, and it could be read to about half this distance. A 1/32-inch hole was drilled at the common center of the circles inscribed on the overlay and a sharp tool which fit tightly into this hole marked the photographic image at its center. These devices are shown in figures 2 and 3.

The factors of interest in these tests were the accuracy to which measurements could be made and the time required to make them. The overlay was centered over the image by hand until readings taken with the shop microscope at four different radial positions indicated that accurate centering had been achieved. The shop microscope was used to read the distance between the edge of the image and the nearest inscribed circle on the overlay. The position of maximum diameter could be determined with the microscope. The maximum diameter was obtained and the center was marked in the minimum time consistent with reasonable accuracy.

The same procedure was used to check the accuracy of the measurements, but considerably more time and care were taken to determine the quantities as accurately as possible. First the transparent overlay was centered accurately over the previously marked centers, and then readings were taken of the image radius at 36 different angular positions. From graphs of these measurements versus angle, such as those shown in figure 4, the maximum diameter and error in positioning of the center marks could be accurately determined.

Measurement Accuracy

The careful measurements of radius are plotted as a function of angular position around the image. Since the photographic image was elliptic, these plots of radius versus angle should be sinusoidal. Due to inaccurate positioning of the center mark, the angle at which the maximum radius occurs on one side is not usually 180° from the maximum on the other side. However, the true maximum radius will always be the highest average value for two readings taken 180° apart. Also shown in figure 4 is the value of maximum image radius measured R_{max} using the normal navigation procedures. The difference in R_{max} between this value and the value indicated by the plotted curve is defined as the error in the measuring technique, $\epsilon_{R_{max}}$. This assumes that the value of R_{max} obtained from the plotted data is the true value of maximum image radius. The difference in radii between two points 180° apart (circle and square symbols) is equal to twice the error in marking the center, plus any error made in repositioning the transparent overlay over the center mark. Since a repositioning error will also occur in the navigation procedure, the sum of both errors can be considered approximately equivalent to the over-all error in positioning the center for the measurement of $\Delta\theta$.

The maximum value of positioning error, $\epsilon_{C_{max}}$, and the value of $\epsilon_{R_{max}}$ are indicated in the figure.

Values $\epsilon_{C_{max}}$ and $2\epsilon_{R_{max}}$ are presented in figure 5 for 10 measurements of images of three different diameters. The magnitude of error does not appear to depend on image diameter. The dashed lines in figure 5 indicate the maximum values for measuring error assumed in the digital computer analysis.

Digital Computer Analysis

A digital computer was programmed to simulate a piloted vehicle returning from the moon using the proposed navigation procedure. Random sets of errors were assumed for the photographic measurements, with the maximum error adjusted to that shown in figure 5. As discussed later, it was considered unnecessary to assume errors in alinement of the vehicle for firing the course correction rockets. For convenience in computer programming, performance of the initial correction procedure was investigated separately from performance of the general correction procedure.

For the initial correction investigation, different sets of initial conditions and of navigational observations were used for a given desired perigee radius and time to arrive at perigee. For each calculated correction, the actual perigee radius and time of arrival were computed, and the differences between these and the desired values were obtained. For each set of initial conditions the navigational errors were computed for 100 different sets of assumed pilot measurement errors. It was assumed that the pilot measurement error had equal probability of being any value between plus and minus the maximum errors obtained in tests of the photographic measuring devices. The results were analyzed on the basis of the maximum navigational error obtained for each 100 sets of assumed pilot errors.

The general correction procedure covered eight additional corrections concerned with achieving the desired perigee radius and not time of arrival. Three sets of initial conditions were assumed for these computations which corresponded to the terminal conditions following the first velocity correction; for each set those cases were used which had maximum error in perigee radius. Again, 100 cases were computed with different sets of assumed pilot errors. The overall navigational error was assumed to be the maximum deviation of perigee radius from the desired perigee radius for the entire 100 cases.

RESULTS AND DISCUSSION

Performance Time

The time required to make the photographic measurements was recorded and indicated that an average of approximately 10 minutes would be required for making the measurements at each measuring station. The time required to perform the required calculations was also determined by performing a number of the calculations described in appendix A and was found to be 15 to 17 minutes. In a human habitation test (to be discussed later), in which two men made a simulated trip to the moon and back, one of the tasks given the subjects was to perform the work required in this navigation procedure. By the end of the 7-day confinement period, the subjects were performing the required measurements and calculations for each general correction in approximately 1 hour. This time is in line with the 1-hour measurement and computing time assumed in the digital simulation.

Initial Correction

Results of numerical calculations of the first velocity correction are presented in table I. The principal quantities of interest are ΔR_{per} and $\Delta\tau$. Here ΔR_{per} is the difference between the desired perigee radius and that which would result from the first correction, and $\Delta\tau$ is the corresponding difference in time of arrival at perigee radius. As discussed previously, the values of ΔR_{per} and $\Delta\tau$ are the maximum values obtained for 100 different sets of random errors imposed on the photographic measurements.

The assumed orbital initial conditions are given in table I. The first value of semimajor axis, $A = 6.6300\times10^{8}$ feet, was chosen (cases A and B) to give an orbit that deviated only slightly from the conditions required to produce the desired values of perigee radius and time of arrival at perigee.

The maximum photographic measuring error, based on an image diameter of about 8 inches and the assumed maximum measuring error in figure 5 of 0.0028 inch, was taken to be a percentage error of 0.035 percent. In the machine calculations, this error was programmed as a direct percentage error in the measurement of range R. The percentage error in $\Delta\theta$ was obtained in the same manner using $\epsilon_{C_{max}} = 0.0013$ inch from figure 5. In this case, however, the film distance corresponding to the angular motion was only about 1/15 of the image diameter as a result of the small angular motion of the vehicle between successive photographic measuring stations. This caused a maximum percentage error of 0.25 in the measurement of $\Delta\theta$.

For cases A and B the assumed error in perigee radius was about 199×10^6 feet and in time of arrival was about 1,387 seconds. Following the first velocity correction the error in perigee radius was only about $\pm 0.5 \times 10^5$ feet and the maximum error in time of arrival was 1,241 seconds, corresponding to a maximum distance of 357 miles on the earth's surface. The largest errors in initial conditions considered (cases E and F), a 503×10^6-foot error in perigee radius and an 18,880-second error in time of arrival, were reduced after the initial velocity correction to errors of no more than 4.5×10^5 feet in perigee radius and 1,027 seconds in time of arrival.

General Velocity Corrections

In the numerical calculations of the general corrections, the same error, 0.035 percent, was assumed in the range measurement. However, the error assumed for $\Delta\theta$ measurements was reduced from 0.25 to 0.05 percent to correspond to the larger film distance through which the image would move. The results in figure 6 show the error in perigee radius after N general corrections were calculated using as the initial values of R, $\dot{R}$, and a the final values after the initial velocity correction for case A in table I. The time increment ΔT between the first two photographic stations was 13,000 seconds. For each subsequent pair of stations ΔT was multiplied by 0.9 to decrease the time between measurements as $\dot{\theta}$ increased. In this way an approximately constant value of $\Delta\theta$ was maintained. Preliminary analysis showed that the final accuracy for a series of corrections was improved when only a fraction, f, of the computed velocity correction was applied to the vehicle at each correction point. Figure 6 was computed with an initial value of $f = 0.75$, which was decreased by 0.07 after each correction. Provision for film-measuring and computing time was made by assuming a time increment of $\Delta t = 1$ hour between the second measuring station and the orbital correction point for each correction. The maximum and minimum perigee radius errors, ΔR_{per}, for each orbital correction for 100 cases are shown.

It is apparent from figure 6 that although the range of perigee radii after each correction is relatively small, the deviation from desired perigee radius becomes large as the number of corrections increases. An examination of these and other similar data indicated that the larger values of ΔR_{per} were due to errors in the method resulting from linearity assumptions. As discussed in appendix A, average values for R_2, $\dot{R}_2$, and $\dot{\theta}_2$ are approximated on the assumption that the vehicle moves in a straight line at constant speed between two adjacent photographic measuring stations. If the distance between measuring stations and the second derivatives of R and θ are small enough, then the error due to this approximation will remain small. Decreasing the distance between measuring stations, however, results in a corresponding decrease in measuring accuracy of

$\Delta\theta$ and decreased accuracy in the computation of $\dot{R}$ and $\dot{\theta}$. The values of $\dot{R}$ and $\dot{\theta}$ change more slowly far from the earth but the problem involved in completing the velocity corrections far from the earth is that only a limited portion of the orbit is available for midcourse corrections.

The factors discussed above were investigated by decreasing Δt, which resulted in completion of the orbital corrections at a distance farther from the earth, and by decreasing the value of ΔT which reduced the distance between adjacent measuring stations. Results of these computations for case A of table I are presented in figure 7. Figures 7(a) and (b) show the results of decreasing Δt to 30 minutes and 15 minutes, respectively. These results show a considerable improvement over those in figure 6, but the effect of the linear approximation discussed previously is still large. Figures 7(c) and (d) show the results of decreasing the initial value of ΔT to 10,865 seconds and 9,320 seconds, respectively. While the approximation error is still apparent, these data indicate an even greater improvement, since the distance between adjacent measuring stations is decreased and the final distance from earth increased.

If the approximation error discussed above could be removed entirely, then the error resulting from this method of navigation would be represented by the difference between maximum and minimum values of ΔR_{per} shown in figures 6 and 7. The magnitude of this difference varies from about 12 miles to a final corridor of approximately 7 miles. An effort was made to decrease this corridor by decreasing the factor f. Figure 8 presents the results of reducing the initial value of f successively to 0.67 and 0.57. For these computations $\Delta T = 10{,}865$ seconds and $\Delta t = 30$ minutes. The reduction in f did not reduce the spread of ΔR_{per} significantly since the final corridor remained between 6 and 7 miles wide. Further decreases in the value of f were not considered feasible since they would decrease the effectiveness of the procedure for large initial errors.

Initial conditions for the data presented in figures 6 through 8 were the orbital quantities following the initial correction shown in case A of table I for the maximum value of ΔR_{per}. The corresponding initial conditions for the minimum value of ΔR_{per}, case B, were used to compute the results presented in figure 9. There is no apparent difference between these results and those in figure 9(b) which used the same values of ΔT, Δt, and f.

The results of this midcourse correction procedure applied to orbits with unduly large initial errors are presented in figure 10. Figure 10(a) shows results computed using the initial conditions resulting after the initial correction in case C of table I; figure 10(b) uses the corresponding initial conditions for case F. For these calculations, $\Delta T = 10{,}865$ seconds, $\Delta t = 30$ minutes, and $f = 0.57$ initially. For these cases the final corridor width is the same as that in figures 8 and 9, but it is apparent from the curves that the value of f is too small to fully overcome the large initial error in ΔR_{per}. In figure 10(a), with an initial large positive error in ΔR_{per}, the effect of the small value of f offsets the linear approximation error discussed previously, resulting in data which look considerably more accurate than for any other case investigated. These results are fortuitous, however, and the results presented in figure 10(b) are a more valid indication of the error involved.

Previous studies of required entry conditions, such as that presented in reference 1, indicate that for a ballistic entry vehicle, the entry corridor is only about 4 miles deep for satisfactory performance. The data in figure 8 show that such a corridor depth probably would not be realized. Reexamination of the computations indicated that a 4-mile corridor might be achieved if the approximation error discussed previously could be nullified. It is not unreasonable to assume that this approximation error could be entirely eliminated in a future development of the proposed navigation system. The crude method studied here assumed the simplest approximations for hand computation. Any development of it would logically include tables, charts, or nomograms, which would give accurate two-body orbital quantities directly from the measurements and perhaps even include perturbation effects from other bodies.

The information in reference 1 indicates that with even a moderate amount of entry lift, the safe entry corridor can be considerably increased. Under the extreme emergency conditions considered herein, a guidance system of the type discussed in reference 2 would not be operative, but preliminary studies indicate that a simple manual guidance procedure can be developed for emergency conditions which will permit an increase in the acceptable entry corridor from 4 miles to something over 20 miles. The present report indicates that for reasonable initial orbital errors an entry corridor well within 20 miles can be achieved even with the crude methods of calculation used in this investigation.

The total fuel weight required for midcourse guidance has been calculated for an assumed vehicle weight of 10,000 lb, a fuel specific impulse of 300 lb-sec/lb, and a total velocity increment obtained by the addition of the initial correction velocity increment to each of the subsequent increments. These data are presented in table II for four of the initial corrections. Also shown are the weights required for an ideal correction at the initial correction point. For reasonable initial errors (cases A and B) the proposed navigation procedure requires at a maximum only about 75 lb more fuel than an ideal correction. With large initial errors, however, there is a much greater increase in fuel requirements.

Confinement Study

In the investigation of the errors associated with this guidance procedure, two-body equations were used in programming the digital computer to determine the vehicle's trajectory. Hence the errors discussed so far do not include perturbations due to the sun and moon. In doing this it was assumed that the trajectory deviation due to these bodies would be canceled by subsequent corrections, with the primary effect being a slight increase in the quantity of fuel used. Subsequent to the main investigation discussed previously, however, an opportunity arose to test the procedure under simulated space flight conditions, and for this investigation a four-body digital program was used to compute the vehicle's trajectory. Results of this study are discussed in the following paragraphs.

The confinement study (ref. 3) was made to explore the physiological and psychological problems which might arise during the long-term confined conditions associated with a circumlunar trip in a small two-man space vehicle. Two men, a research pilot and a physiologist, were placed in such a capsule and remained there continually for a period of 7 days. In addition to a battery of physiological and psychological tests performed during this time, an average of about 45 minutes out of every 4 hours was devoted to performing the navigation tasks under discussion here. To avoid undue complexity for the capsule occupants the initial correction procedure was neglected, and only the general correction procedure tasks performed by them. The subjects were familiarized with the various tasks beforehand but had no time to become adept at performing them.

A digital computer program employing four-body equations was used to compute the vehicle's trajectory from an assumed set of initial conditions.

Photographs were passed to the confined subjects with a value for angular separation of the stars from which they could compute the magnification factor by measuring the star separation. The subjects then made the navigation measurements and calculations using the equipment shown in figures 2 and 3.

Two complete returns to earth were performed during the confinement period, and perhaps the most interesting result obtained was the number and effect of mistakes in arithmetic. Measurement errors of the types previously discussed will result in relatively small trajectory errors that can be canceled by subsequent corrections. However, arithmetic mistakes in the required calculations can result in trajectory errors greater by orders of magnitude. During the first return to earth arithmetic mistakes were retained and at the end it was obvious that the navigation procedure was useless unless some means of eliminating these mistakes was devised. For the second return trajectory, all errors in measurement were retained in the calculations, but all arithmetic mistakes were corrected before proceeding with the orbital computations. Using this basis of operation, six orbital midcourse corrections were made and the error in perigee radius was reduced from 500 to 5.7 miles. The fuel used for all corrections, based on the same specific impulse and weight assumptions used previously, was 200 lb. These values can be compared to an initial error in perigee radius of 500 miles, and the fuel required for an ideal correction at the initial conditions, 84 lb.

Further Considerations

Two factors that require additional comments are the effect of misalinement of the course correction rockets and difficulties involved in obtaining the required photographs.

Rocket misalinement errors are of two types: velocity errors in the orbital plane, and velocity errors perpendicular to the orbital plane. Errors perpendicular to the orbital plane will cause minor changes in the orbital plane angle, which can easily be tolerated in the over-all mission. Errors in the orbital plane will affect the orbital perigee radius, but this effect will be negligible. Generally the same type of effect will result as from changes in the factor f.

An extensive development program will be required to produce the necessary photographic equipment. Accuracy of photographic measurements might be seriously decreased if a full-disk earth image were not obtained. Therefore, photographic emulsions and filters will have to be developed to function in special radiation bands. Also, since a large intensity difference will exist between earth and background stars, special procedures must be developed for handling the problem of obtaining both earth and star images on the same photographic plate.

CONCLUDING REMARKS

The navigational procedure presented herein could provide a safe entry into the earth's atmosphere if some aerodynamic lift were available. However, the procedure is not acceptable in its present form, since no attempt has been made to optimize any phase of it. This study has merely been an effort to show that even with the crudest type of system, the general procedure suggested is capable of doing the required job.

Before a reasonable final navigation system, based on this general procedure, could be formulated, extensive additional optimization studies must be made. It appears necessary both from performance time requirements and accuracy requirements to utilize some system of graphs, tables, or nomograms to solve the orbital equations instead of the approximate equations assumed in this investigation. Also, considerable possibility exists for improvement in the required time and accuracy of making photographic image measurements. It may even be more desirable to obtain a different set of primary measurements and deduce orbital quantities by some other procedure such as astronomical triangulation. It should be remembered, however, that the emergency systems considered here are based on the assumption that no electronic devices are available.

It will be necessary to investigate factors such as discussed above before a navigation system can be proposed which is satisfactory from the standpoint of relative effectiveness and simplicity. However, it is pertinent to note that, even without these additional studies, the crude techniques used in this investigation give results which indicate the possibility of safely returning a lifting entry vehicle from the moon.

Ames Research Center
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APPENDIX A

GENERAL CORRECTION PROCEDURE

A correction velocity is determined by first computing the orbital parameters using values of range, R, range rate, $\dot{R}$, and angular rate, $\dot{\theta}$, at some point on the orbit. These values are obtained from photographs taken of the earth at two points, points 1 and 3 in figure 1. The photographs, which provide as large an image of the earth as is feasible, are used to determine range at each point and the net angular position change, $\Delta\theta$. (For accuracy estimations in this investigation, an image 8 inches in diameter was assumed.) The photographs will be used to determine angular rate at point 2 in figure 1, and will contain not only an image of the earth at points 1 and 3, but also at least two of the same stars in the background of both photographs. When the photographs are alined so that the two stars are superimposed, then the orbital angular change, $\Delta\theta$, is given by the distance between the centers of the two earth images. Photographs taken through an optical filter system which transmits only in the radiation range of the H_2O or CO_2 molecule at earth temperatures or some similar selective frequency would show the complete earth disk extended to the tropopause and would minimize atmospheric diffraction effects. The optical and photographic system must not only provide a complete earth disk image but must also be sufficiently sensitive to stellar radiation to show star images near the earth image.

Given the photographs and the time increment between taking them at points 1 and 3, approximate values for range R, range rate $\dot{R}$, and angular rate $\dot{\theta}$ are computed for point 2 in the following manner:

$$R_1 = \frac{R_e}{\tan(\alpha d_1/2l)}$$

$$R_3 = \frac{R_e}{\tan(\alpha d_3/2l)}$$

$$\Delta\theta = \left(\frac{D}{l}\right)\alpha$$

$$R_2 = \frac{R_1 + R_3}{2}$$

$$\dot{R}_2 = \frac{R_3 - R_1}{\Delta T}$$

$$\dot{\theta}_2 = \frac{\Delta\theta}{\Delta T}$$

where

d_1 diameter of earth's image on film at point 1

d_3 diameter of earth's image on film at point 3

D distance between centers of images at points 1 and 2

l distance separating images of calibration star pairs

α known angular separation of calibration star pairs

ΔT time increment between points 1 and 3

R_e known radius of the earth tropopause, 0.20986×10^8 ft

From R_2, $\dot{R}_2$, and $\dot{\theta}_2$, the orbital perigee radius R_{per} is computed in the following manner:

$$V_2{}^2 = \dot{R}_2{}^2 + R_2{}^2\dot{\theta}_2{}^2$$

$$A = \frac{1}{(2/R_2) - (V_2{}^2/\mu)}$$

$$e = \sqrt{\left(1.0 \frac{R_2}{A}\right)^2 + \frac{R_2{}^2\dot{R}_2{}^2}{\mu A}}$$

$$R_{per} = A(1 - e)$$

where

V_2 orbital velocity at point 2

A orbital semimajor axis

μ gravitational constant for earth, 1.40773×10^{16} ft^3/sec^2

e orbital eccentricity

Finally, ΔR_{per}, the difference between R_{per} and the desired value of perigee radius, is computed. With values of R_2, e, and ΔR_{per} the required orbital correction velocity at point 2 can be obtained from curves such as that shown in figure 11, which is taken from reference 1. (The abscissa is R_2 normalized with respect to the distance at the time of the mission between earth and moon, R_m.) For the sake of simplicity, this value is used as the correction at point 4 of figure 1.

APPENDIX B

INITIAL CORRECTION PROCEDURE

Since orbital inaccuracies are more sensitive to guidance errors at positions far removed from the earth, and because it is desirable to have control over time to perigee from some point on the orbit, a more complex procedure for computing orbital parameters was adopted for the first velocity correction. The first correction is most suitable for this purpose because the low value of $\dot{\theta}$ makes it necessary to allow about 10 hours between points 1 and 3 in order to obtain sufficiently large values of $\Delta\theta$ for accurate measurement.

In order to achieve greater accuracy 11 sets of photographs are taken, spaced about 45 minutes apart, instead of 2 sets as discussed in appendix A. With each adjacent pair of measuring stations considered as being points 1 and 3 in figure 1, the orbital angular momentum per unit mass, H, is computed at the intermediate point 2 from the equation

$$H = R_2{}^2\dot{\theta}_2$$

where R_2 and $\dot{\theta}_2$ are obtained as in appendix A. For a Keplerian orbit, H is a constant. After the eleventh measurement is made, the arithmetic average value $\overline{H}$ is obtained. This value is used to correct other measured quantities. With the assumption that the error in H at any point is produced by equal percentage errors in R_2 and $\dot{\theta}_2$, it is found that the corrections for R and H are related by

$$R_1' = \sqrt{\left[1 + \frac{2}{3}\left(\frac{\overline{H}}{H} - 1\right)\right] R_1{}^2}$$

$$R_3' = \sqrt{\left[1 + \frac{2}{3}\left(\frac{\overline{H}}{H_3} - 1\right)\right] R_3{}^2}$$

where a prime indicates a corrected value. Then the following computations are made in order to arrive at a corrected value for orbital semimajor axis, A',

$$R_2' = \frac{R_1' + R_3'}{2}$$

$$\dot{\theta}_2' = \frac{\overline{H}}{(R_2')^2}$$

$$\dot{R}_2' = \frac{R_2' - R_1'}{\Delta T}$$

$$A' = \frac{1}{\frac{2}{R_2'} - \frac{(\dot{R}_2')^2 + (R_2'\dot{\theta}_2')^2}{\mu}}$$

The time interval between point 2 and point 4 at which the initial velocity correction will be made is then selected and conditions at point 4 are computed in the following manner:

$$R_4 = R_2 + \dot{R}_2'(T_4 - T_2)$$

$$\dot{\theta}_4 = \frac{\overline{H}}{R_4^2}$$

$$\dot{R}_4 = -\sqrt{\mu\left(\frac{2}{R_4} - \frac{1}{A'}\right) - (R_4\dot{\theta}_4)^2}$$

In order to control the earth longitude of the perigee radius position, the orbital true anomaly, θ, is first obtained from the equation

$$\theta_4 = \cos^{-1}\left\{\frac{1}{e}\left[\frac{A'}{R_4}(1 - e^2) - 1\right]\right\}$$

where

$$e^2 = \left(1 - \frac{R_4}{A'}\right)^2 + \frac{(R_4\dot{R}_4)^2}{\mu A'}$$

Given θ_4, the rotational period of the earth, and a landmark photograph of the earth, the time τ for the vehicle to move from R_4 to R_{per} can be computed. To simplify the present study a number of cases was computed numerically to obtain the following approximate equation:

$$\left(\frac{\dot{R}_4}{R_4\dot{\theta}_4}\right) = 28.52 - 21.28 \tan^{-1}\left[\frac{\tau - \left(1 - \frac{0.75\times10^9}{R_4}\right)(1.25\tau + 50{,}000)}{47{,}000}\right]$$

The above equation was obtained empirically from computed data, for $R_{per} = 0.20986\times10^8$ feet, and if the exact value of $(\dot{R}_4/R_4\dot{\theta}_4)$ is used, then the time to perigee, τ, will be correct within approximately 500 seconds for the following ranges:

$$0.65\times10^9 \leq R \leq 0.85\times10^9$$

$$110{,}000 \leq \tau \leq 240{,}000 \qquad \text{for} \quad R = 0.85\times10^9$$

$$80{,}000 \leq \tau \leq 160{,}000 \qquad \text{for} \quad R = 0.65\times10^9$$

Having computed the actual and the desired value of τ, one then calculates the desired value of $(\dot{R}_4/R_4\dot{\theta}_4)$; a correction to the actual velocity $R_4\dot{\theta}_4$ provides this desired velocity ratio. The corresponding R_p is computed as in appendix A. A further correction to velocity $R_4\dot{\theta}_4$ is then computed to obtain the desired R_{per}, but in this case a correction to velocity $\dot{R}_4$ is also assumed to maintain the above desired velocity ratio. Since $\dot{R}_4$ has been changed, the correction velocity for correcting R_{per} will have to be recalculated a second or third time before both R_{per} and τ have the desired values.

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3. Rathert, George A., McFadden, Norman M., Weick, Richard F., Rogers, Terence A., Patton, Mark R., and Stinnett, Glen W.: Minimum Crew Space Habitability for the Lunar Mission. Paper presented at the Bioastronautics Session at the 17th Annual Meeting of American Rocket Society and Space Flight Exposition, Los Angeles, Calif., Nov. 13-18, 1962.

TABLE I.- ORBITAL CONDITIONS BEFORE AND AFTER INITIAL CORRECTION

	Case A	Case B	Case C	Case D	Case E	Case F
Initial conditions						
$R = 8.73101 \times 10^8$ ft and $\dot{R} = -3058$ ft/sec for all cases						
a, ft	6.36858×10^8		12.7370×10^8		25.4740×10^8	
e	0.965781		0.704623		0.794216	
ΔR_{per}, ft	8.0678×10^5		35.524×10^7		50.323×10^7	
$\Delta \tau$, sec	1387		39,000		18,880	
Final Conditions						
R, ft	7.7182×10^8		7.7461×10^8		7.7291×10^8	
A, ft	6.6300×10^8	6.6958×10^8	6.9680×10^8	6.8637×10^8	7.2388×10^8	7.1296×10^8
e	0.968271	0.968740	0.969555	0.969952	0.970642	0.971204
ΔR_{per}, ft	0.5016×10^5	-0.5491×10^5	2.2810×10^5	-3.6168×10^5	2.6558×10^5	-4.5566×10^5
$\Delta \tau$, sec	1241		1079		1027	
$\delta \dot{R}$, ft/sec	122	149.9	692.8	656.7	1043.3	1010.1
$\delta (R\dot{\theta})$, ft/sec	17.0	19.4	2888.9	2902.7	3714.5	3731.4

Desired perigee radius 2.09859×10^7 ft

TABLE II.- REQUIRED VELOCITY INCREMENTS AND FUEL WEIGHTS FOR MIDCOURSE GUIDANCE

	Initial correction case (table I)			
	A	B	C	D
Figure	6(b)	7	8(a)	8(b)
Initial correction velocity increment, ft/sec	127	169.3	3582	4742
Total velocity increment for general corrections, ft/sec	2.97	0.95	6.95	11.56
Total velocity increment, ft/sec	130	170	3589	4753
Required fuel weight, lb	135.6	177.6	4505	6367
Velocity increment for ideal correction, ft/sec	98.5	98.5	2968	3854
Ideal correction fuel weight, lb	102.5	102.5	3600	4908
Excess fuel required for manual guidance procedure, percent	32.3	73.2	25.2	29.7

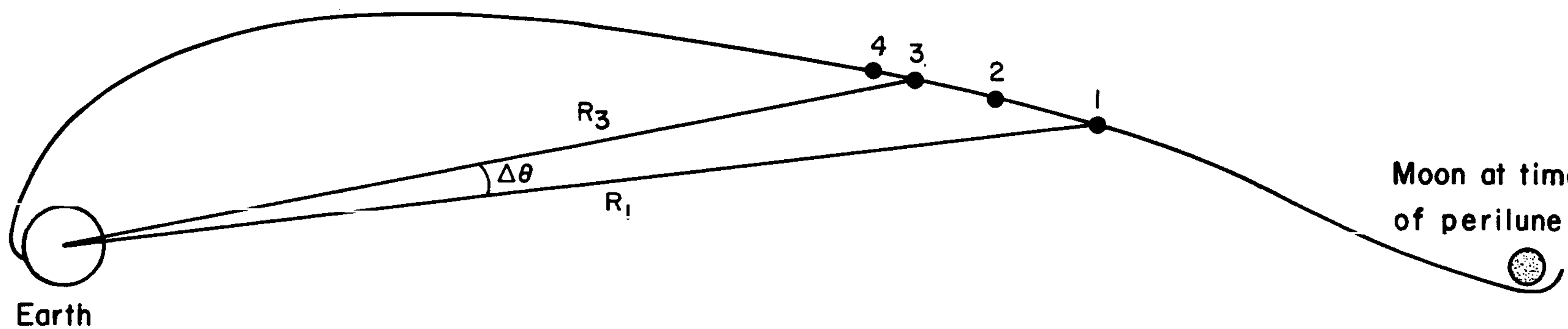


Figure 1.- Return orbit from the moon.

A-29485

Figure 2.- Image measuring apparatus.

A-29486

Figure 3.- Measuring apparatus in position for obtaining $\Delta\theta$.

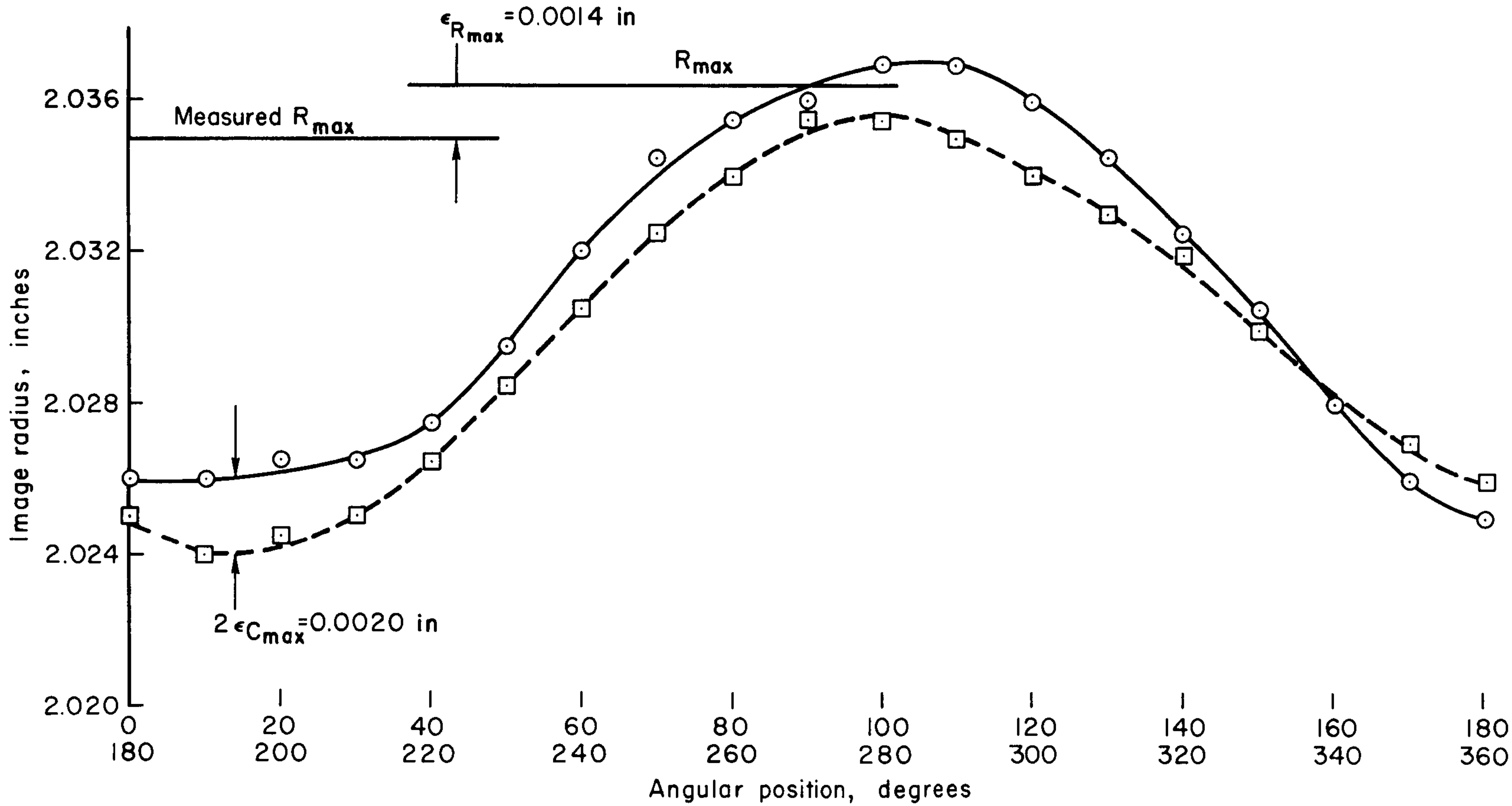


Figure 4.- Variation of image radius with angular position.

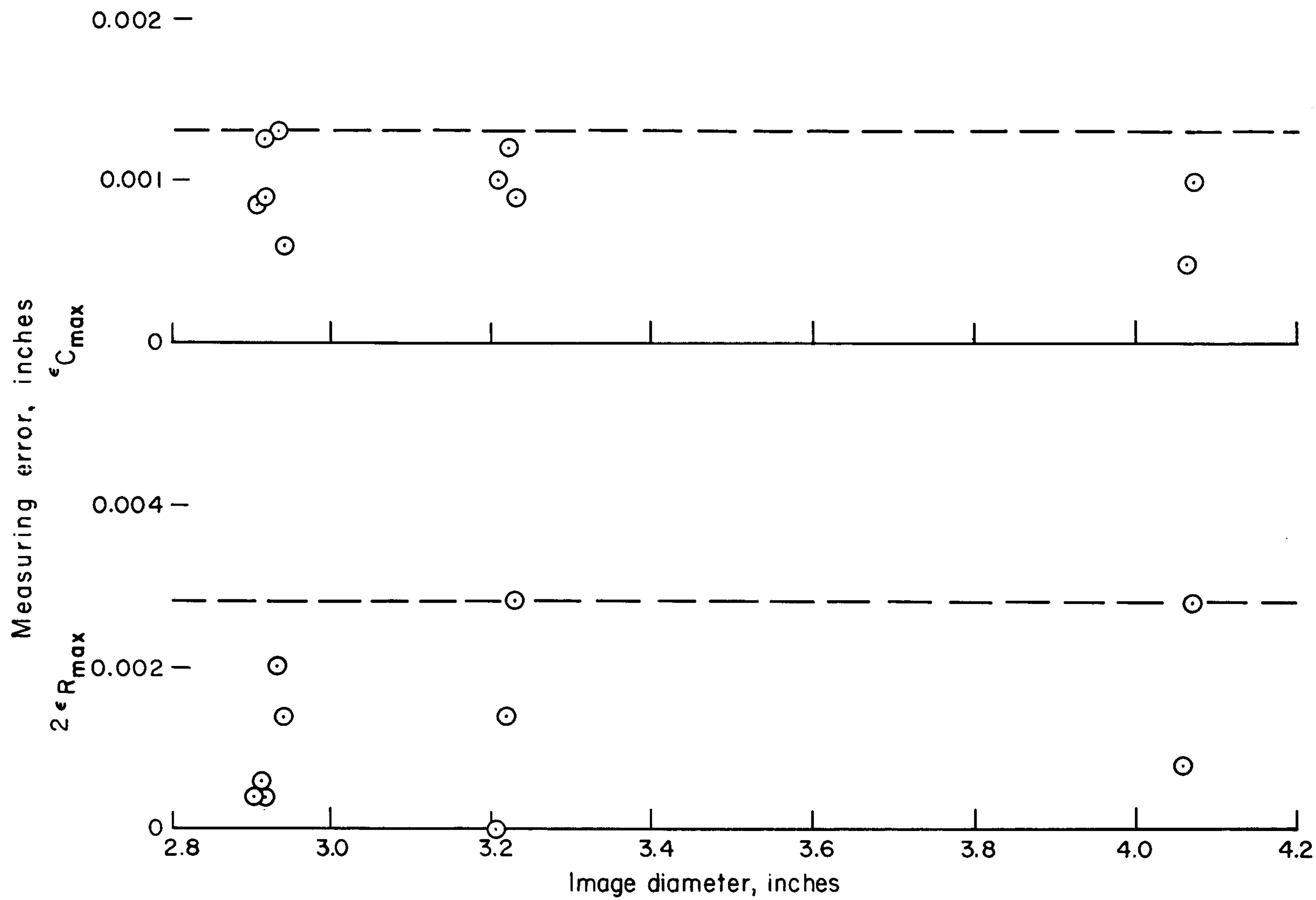


Figure 5.- Variation of measuring error with image diameter.

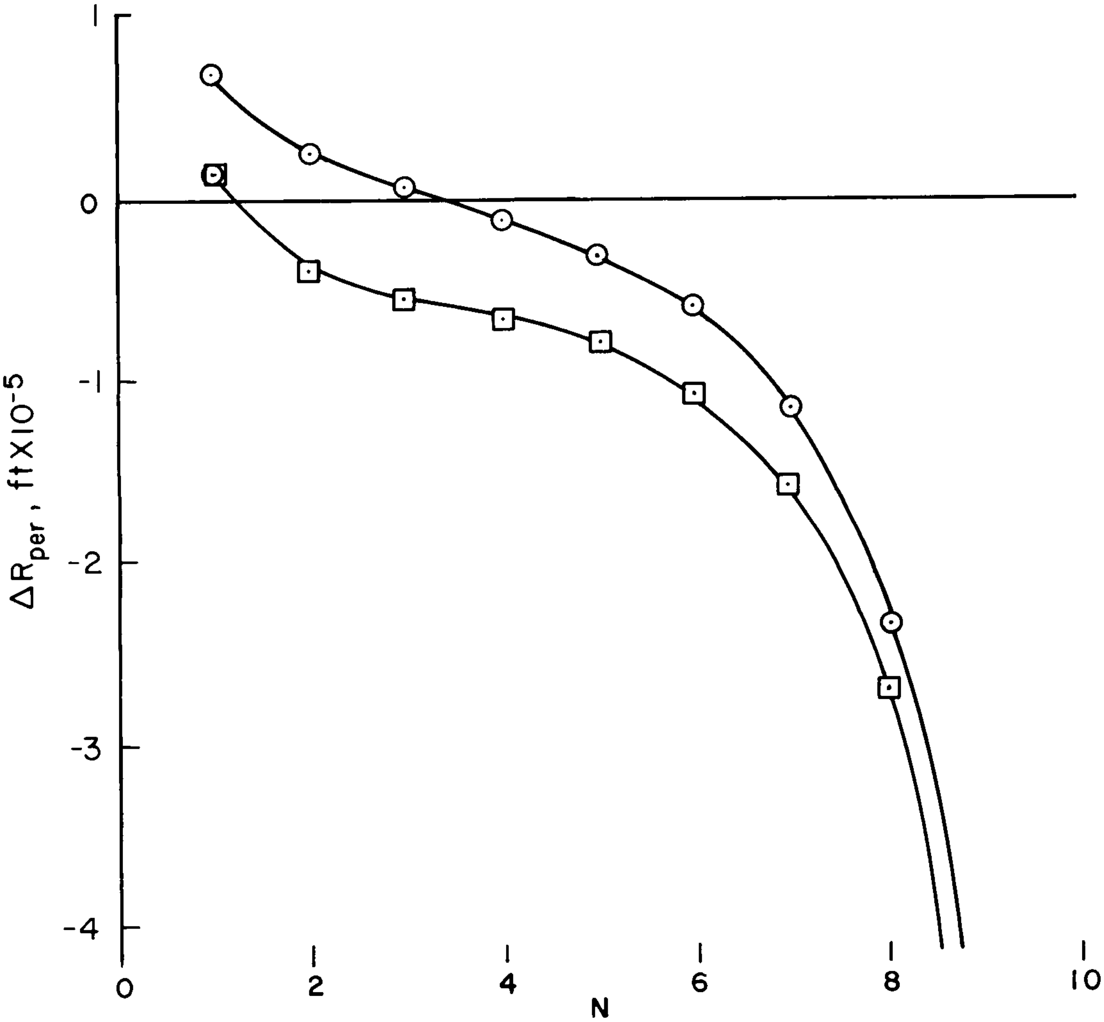
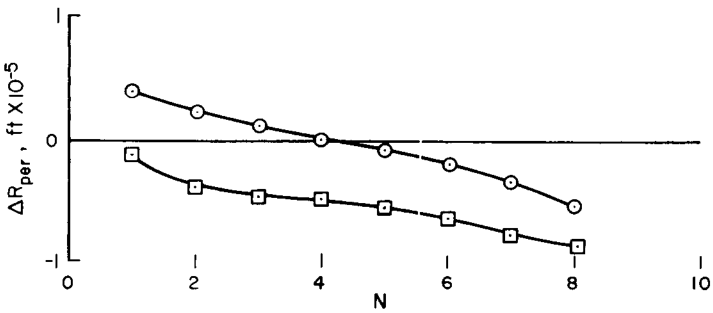
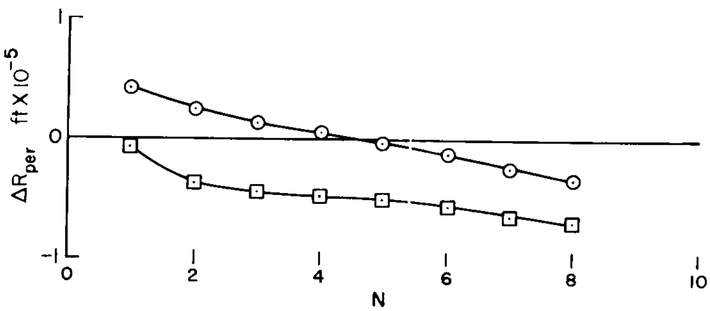


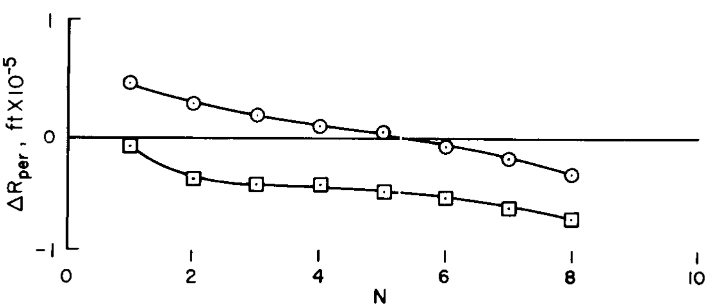
Figure 6.- Error in perigee radius after N corrections.



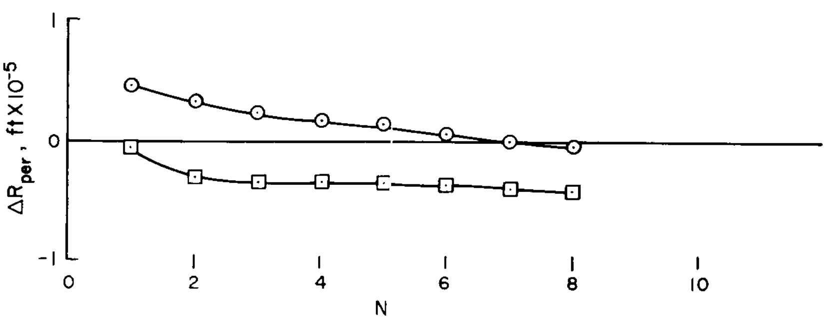
(a) Δt = 30 minutes; ΔT = 13,000 seconds



(b) Δt = 15 minutes; ΔT = 13,000 seconds

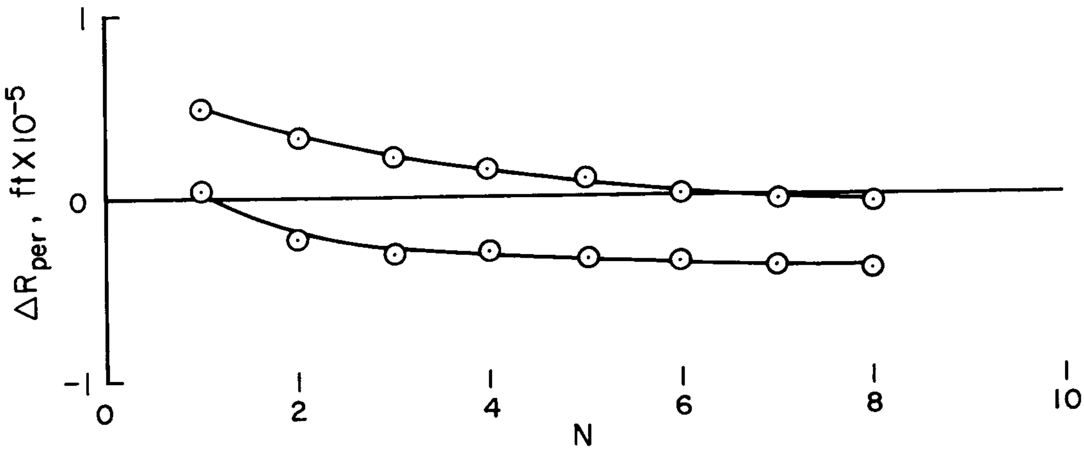


(c) Δt = 1 hour; ΔT = 10,865 seconds

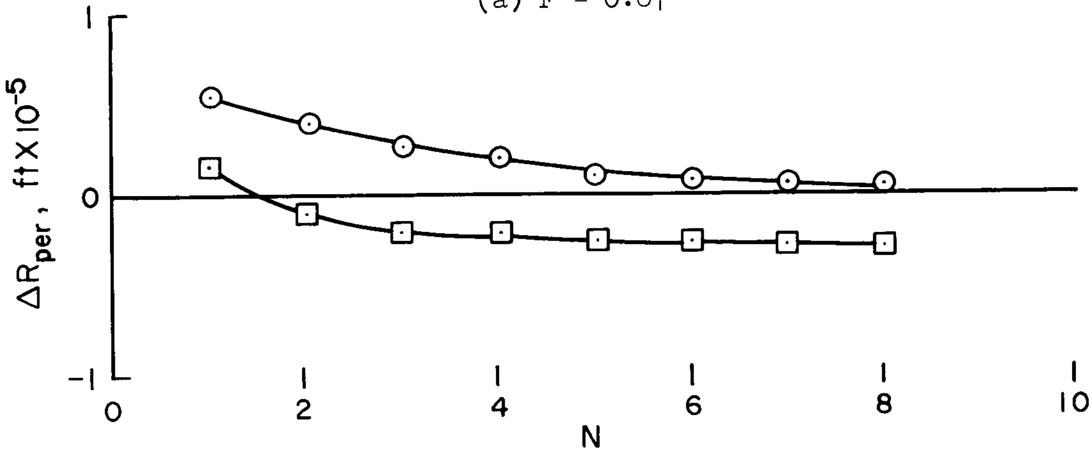


(d) Δt = 1 hour; ΔT = 9,320 seconds

Figure 7.- The effect of Δt and ΔT on perigee radius error.



(a) F = 0.67



(b) f = 0.57

Figure 8.- The effect of f on perigee radius error.

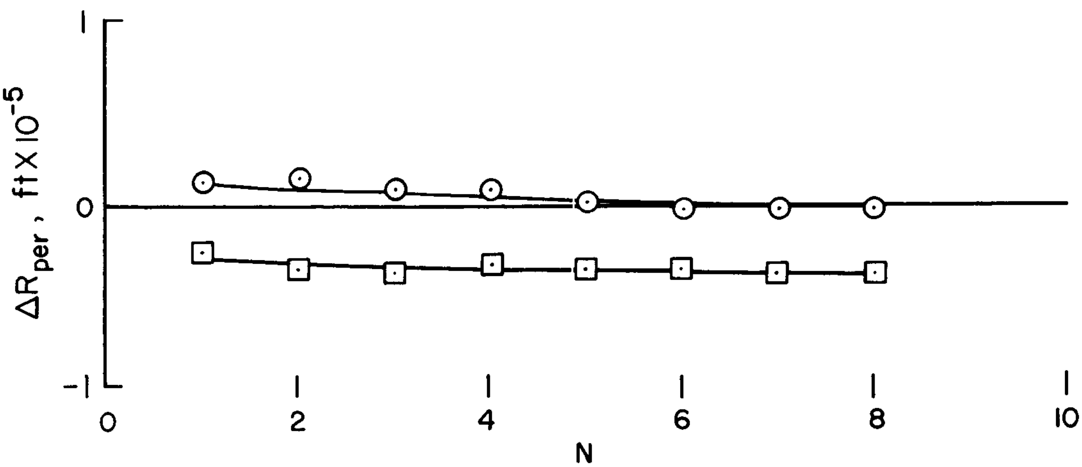
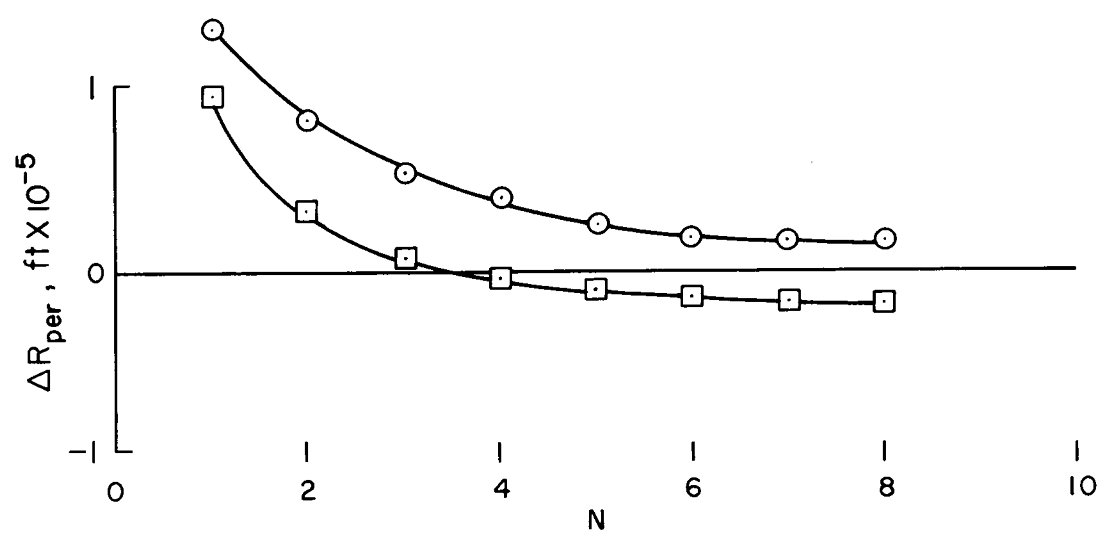
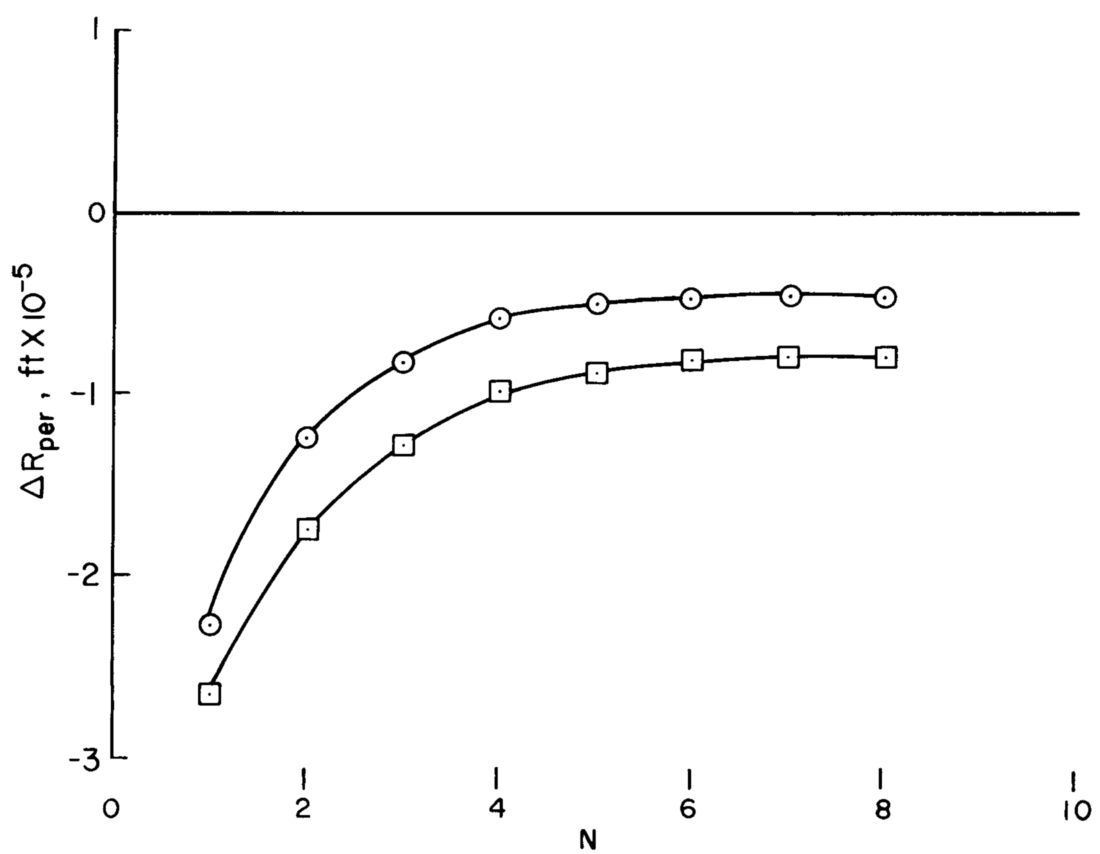


Figure 9.- The error in perigee radius after N corrections.



(a) Large initial positive error.



(b) Large initial negative error.

Figure 10.- The effect of large initial orbital errors on perigee radius error.

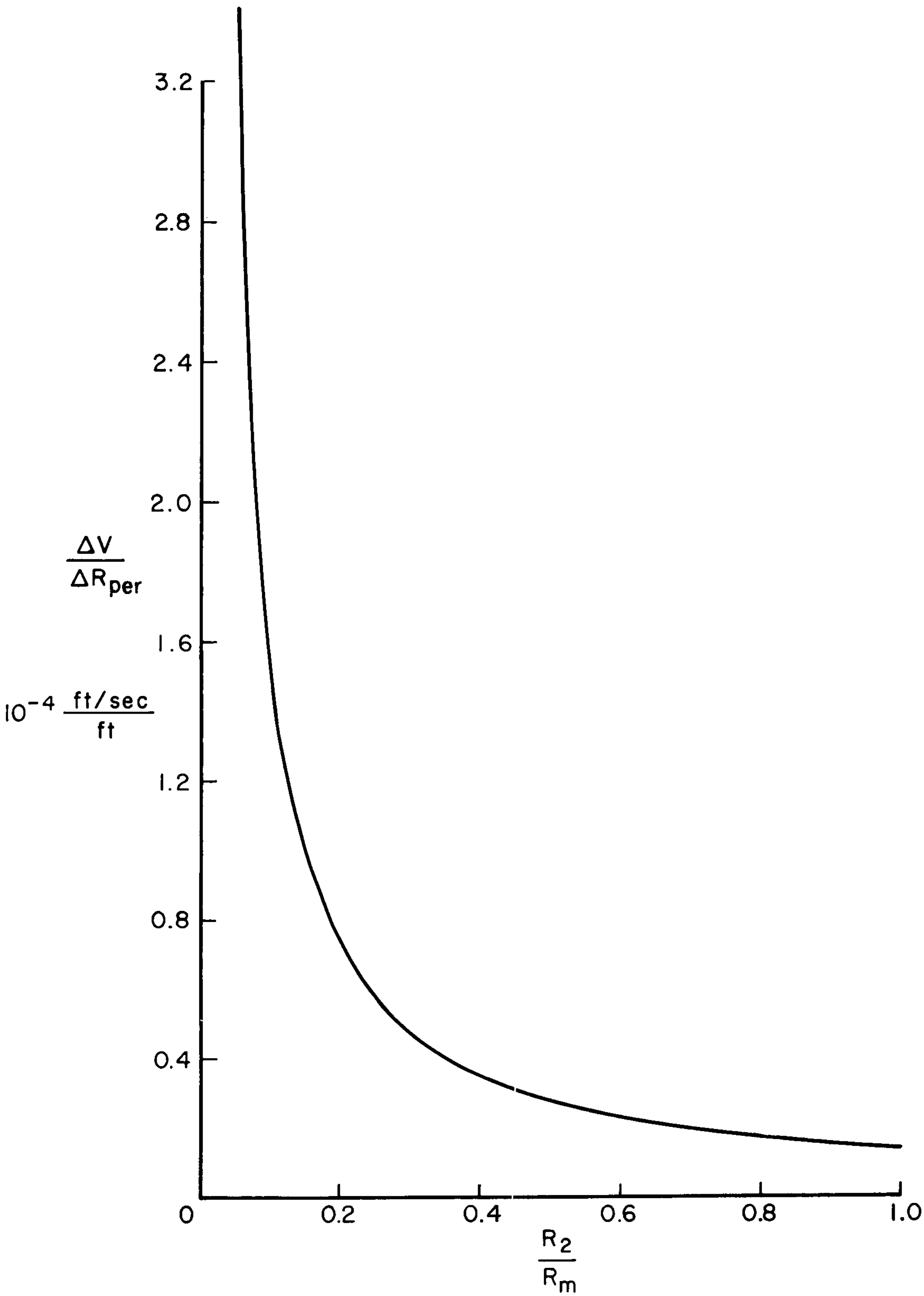


Figure 11.- Required orbital correction velocity.